

Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/31**

Paper 3 Further Mechanics

May/June 2026

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme contains the instructions and marking guidance that our examiners used to mark candidate responses for this component.

Before they start marking, examiners complete a standardisation process. They review a range of candidate responses and discuss the mark scheme in detail to agree which answers can and cannot be accepted. Examiners award marks where it is clear that candidate responses have met the requirements of the mark scheme. These requirements may vary between different components or different versions of the same component, depending on the exact requirements of the question and the candidate responses seen during the standardisation process.

Sometimes, our mark schemes may allow marks to be awarded to responses which contain elements that are factually incorrect or for content not covered by the syllabus. We do this in response to the demands of a specific question to support candidates and make sure that our assessments are fair. However, these allowances should not be used as a guide for teaching and learning.

We cannot discuss the mark scheme with schools, and we recommend that schools read the mark scheme together with the assessment material and the Principal Examiner Report for Teachers.

This document consists of **22** printed pages.

General marking principles

All examiners must apply these marking principles when marking candidate responses.

1	Examiners must award marks for each response in line with: - the specific content defined in the mark scheme - the specific skills defined in the mark scheme or in the level descriptions - the standard of response required as exemplified by the standardisation scripts.
2	Examiners must award whole marks (not half marks, or other fractions).
3	Examiners must award marks positively . This means that: - marks are not deducted for errors or omissions - marks are awarded for correct/valid answers as defined in the mark scheme and also for other valid answers which go beyond the scope of the syllabus and mark scheme referring to your team leader as appropriate - the quality of spelling, punctuation and grammar are judged only when the mark scheme indicates that these features are specifically assessed in the question. However, the meaning of all responses must be unambiguous.
4	Examiners must consistently apply the requirements of the mark scheme and any rules for what to do when candidates have not followed instructions correctly.
5	Examiners must use the full range of marks defined in the mark scheme, unless limited by the quality of the candidate responses seen.
6	Examiners must award marks according to the mark scheme and must not award marks with grade thresholds or grade descriptions in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance												
1(a)	$\left(\frac{1}{2}x\right)\times\pi r^2x+\left(\frac{9}{8}r+x\right)\times18\pi r^3=\bar{x}\times\left(\pi r^2x+18\pi r^3\right)$	M1	Moments equation with three terms, dimensionally correct so must be of the form $\alpha r^2x^2+\left(\beta r+\gamma x\right)r^3=\bar{x}\times\left(\lambda r^2x+\mu r^3\right)$ where $\alpha,\beta,\gamma,\lambda,\mu$ are all non-zero constants.												
			For reference:												
			<table><tr><th>Shape</th><th>Distance</th><th>Volume</th></tr><tr><td>Cylinder</td><td>$\frac{1}{2}x$</td><td>πr^2x</td></tr><tr><td>Hemisphere</td><td>$\frac{3}{8}\times3r+x$</td><td>$\frac{1}{2}\times\frac{4}{3}\pi(3r)^3=18\pi r^3$</td></tr><tr><td>Total</td><td>$\bar{x}$</td><td>$\pi r^2(x+18r)$</td></tr></table>	Shape	Distance	Volume	Cylinder	$\frac{1}{2}x$	πr^2x	Hemisphere	$\frac{3}{8}\times3r+x$	$\frac{1}{2}\times\frac{4}{3}\pi(3r)^3=18\pi r^3$	Total	$\bar{x}$	$\pi r^2(x+18r)$
			Shape	Distance	Volume										
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Total	$\bar{x}$	$\pi r^2(x+18r)$													
		B1	At least two terms correct.												
		A1	All correct.												
	$\bar{x}=\frac{2x^2+81r^2+72xr}{4x+72r}$	A1	OE, for example, $\frac{\frac{1}{2}\pi x^2r^2+\frac{81}{4}\pi r^4+18\pi r^3x}{\pi r^2x+18\pi r^3}$. All individual terms must be simplified. ISW after correct answer seen.												
		4													

Question	Answer	Marks	Guidance
1(b)	$\tan \theta = \frac{\bar{x} - x}{3r}$	*M1	For $\tan \theta = \frac{\bar{x} - x}{3r}$ or $\tan \theta = \frac{x - \bar{x}}{3r}$ or $\tan \theta = \frac{3r}{\bar{x} - x}$ or $\tan \theta = \frac{3r}{x - \bar{x}}$ only (allow equivalents with x replaced with r or vice versa).
	$\tan \theta = \frac{\frac{2x^2 + 81x^2 + 72x^2}{4x + 72x} - x}{3x}$	DM1	For $\tan \theta = \frac{\bar{x} - x}{3x}$ or $\tan \theta = \frac{x - \bar{x}}{3x}$ only with <i>their</i> $\bar{x}$ correctly substituted – <i>their</i> $\bar{x}$ must be dimensionally correct and have come from an expression of the form $\frac{ax^2 + br^2 + cxd}{dx + er}$ with a, b, c, d and e being non-zero. Must be in terms of either x or r only. If correct, may see $\frac{155x}{76} - x = \frac{79}{76}x$ in numerator. For reference: $\tan \theta = \pm \frac{79}{228}$.
	$\theta = 19.1^\circ$	A1	AWRT 19.1 (19.110752...) or 0.334 radians (0.3335455...). Allow $\tan \theta = -\frac{79}{228} \Rightarrow \theta = 19.1^\circ$ for full marks (but A0 if given as a negative angle).
		3	

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Question	Answer	Marks	Guidance
2	$(F_{\text{radial}} =) 7 \times 12 \times 4^2 [=1344]$	B1	Correct unsimplified expression for the radial force.
	Attempt to apply N2L either horizontally or vertically to form an equation.	M1	Correct number of terms, dimensionally correct, allow sign errors and sin/cos mix only. Values of sin/cos must be substituted. Allow 37 or better (36.8698...) for the angle between AP and the horizontal and 23 or better (22.6198...) for the angle between BP and the horizontal.
	$\frac{3}{5}T_A = \frac{5}{13}T_B + 7g \quad (\Rightarrow 39T_A = 25T_B + 4550)$	A1	Allow 37 or better (36.8698...) for the angle between AP and the horizontal and 23 or better (22.6198...) for the angle between BP and the horizontal, for example, $T_A \sin(37) = T_B \sin(23) + 7g$.
	$\frac{4}{5}T_A + \frac{12}{13}T_B = 7 \times 12 \times 4^2 \quad (\Rightarrow 13T_A + 15T_B = 21840)$	A1	Allow 36 or better (36.8698...) for the angle between AP and the horizontal and 23 or better (22.6198...) for the angle between BP and the horizontal, for example, $T_A \cos(37) + T_B \cos(23) = 7 \times 12 \times 4^2$.
	$T_A = 675$ and $T_B = 871$	A1	AWRT 675 and 871 only.
		5	

Question	Answer	Marks	Guidance
3(a)	$8 - 0.5v^2 = 6.4v \frac{dv}{dx}$	*B1	Correct use of Newton's second law – acceleration must be $v \frac{dv}{dx}$.
	$x = \int \frac{6.4v}{8 - 0.5v^2} dv = -6.4 \ln 8 - 0.5v^2 + c$ or $x = -\int \frac{6.4v}{0.5v^2 - 8} dv = -6.4 \ln 0.5v^2 - 8 + c$	*M1	Separate variables correctly (from a differential equation of the form $\alpha + \beta v^2 = \gamma v \frac{dv}{dx}$) and obtain an equation of the form $x = k_1 \ln(k_2 + k_3 v^2)$ where k_1, k_2 and k_3 are non-zero constants OR $x = A \ln(k_1 + k_2 v) + B \ln(k_3 + k_4 v)$ where A and B must have the same sign. Condone lack of $+c$.
	$x = -6.4 \ln 8 - 0.5v^2 + c$ (*) or $x = -6.4 \ln 0.5v^2 - 8 + c$ (**)	A1	OE, for example, $-\frac{1}{2} \ln(4 - v) - \frac{1}{2} \ln(4 + v) = \frac{x}{12.8} + c$. Condone lack of modulus signs in (*). However, if solving $x = -\int \frac{6.4v}{0.5v^2 - 8} dv$ then modulus must be included in (**). Condone lack of $+c$.
	$x = 0, v = 0 \Rightarrow c = 6.4 \ln 8$	DM1	Use correct initial conditions to find c . Note that $0 = -6.4 \ln(0 - 8) + c$ leading to a value of c is DM0 .
	1.37 m s^{-1}	A1	WWW AWRT 1.37 (1.3711489...) – or exact, for example, $v = 4\sqrt{1 - e^{-\frac{1}{8}}}$.
		5	

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Question	Answer	Marks	Guidance
3(b)	$8 - 0.5v^2 = 6.4 \frac{dv}{dt}$	B1	Correct use of Newton's second law – acceleration must be $\frac{dv}{dt}$.
	$t = \int \frac{12.8}{16 - v^2} dv = 1.6 \ln \left \frac{4 + v}{4 - v} \right + c$	M1	Separate variables correctly (from a differential equation of the form $\alpha + \beta v^2 = \gamma \frac{dv}{dt}$) and obtain an equation of the form $t = A \ln(k_1 v + k_2) + B \ln(k_3 v + k_4)$ where A, B, k_1, k_2, k_3 and k_4 are non-zero constants but A and B must have different signs. Allow $t = \lambda \operatorname{artanh}(\mu v)$ for any $\lambda, \mu \neq 0$ where $\mu \neq \pm 1$.
	$t = 1.6 \ln \left \frac{4 + v}{4 - v} \right + c (*)$ or $t = -1.6 (\ln v - 4 - \ln v + 4) + c (**)$	A1	Allow unsimplified and $t = 3.2 \operatorname{artanh}(\frac{1}{4}v) + c$. Condone lack of $+c$. Condone lack of modulus signs in (*), but in (**) must see modulus sign on at least the $\ln(v - 4)$ term.
	Find c : $t = 0$ when $v = 0 \Rightarrow c = 0$ $t = 1.6 \ln \left \frac{4 + v}{4 - v} \right (*)$ or $t = 1.6 \ln \left \frac{4 + v}{v - 4} \right (**)$	A1	As a minimum must state, ' $t = 0, v = 0 \Rightarrow c = 0$ '. ISW if re-arranged to make v the subject. Allow as two separate log terms. Allow $t = 3.2 \operatorname{artanh}(\frac{1}{4}v)$. Condone lack of modulus sign for (*) but required in (**) in at least the $\ln(v - 4)$ term. If answer is in terms of x , for example $t = 1.6 \ln \left \frac{4 + x}{4 - x} \right $, then maximum B1 M1 A0 A0 possible. Allow recovery for full marks.
		4	

Question	Answer	Marks	Guidance
4(a)	$v_1 = \frac{12}{13}eu$ and $v_2 = \frac{5}{13}u$ or $v \sin \beta = \frac{12}{13}eu$ and $v \cos \beta = \frac{5}{13}u$	B1	v_1 is the velocity component of P perpendicular to the barrier after impact and v_2 is velocity component of P parallel to the barrier after impact. May see LHS of v_1 equation as $v \sin \beta$ or $v \cos \beta$ and similar for v_2 , but must have opposite trigonometric ratios. Allow $v \sin \beta = eu \times \sin(67)$ or better and $v \cos \beta = u \times \cos(67)$ or better.
	$\frac{1}{2}mv^2 = \frac{4}{13} \times \frac{1}{2}mu^2 \Rightarrow v^2 = \frac{4}{13}u^2$	*B1	Correct equation relating kinetic energies. This mark can be implied by seeing $v^2 = \frac{4}{13}u^2$.
	$\left(\frac{12}{13}eu\right)^2 + \left(\frac{5}{13}u\right)^2 = \frac{4}{13}u^2$	DM1	Correct use of $v^2 \sin^2 \beta + v^2 \cos^2 \beta = v^2$ to form an equation in e (and possibly u) only. Allow minor slips, for example, e rather than e^2 but other terms must have been squared. One component must be either $\frac{12}{13}eu$ or $\frac{5}{13}eu$ and the other must be either $\frac{5}{13}u$ or $\frac{12}{13}u$ (or using $\sin(67)$, $\cos(67)$, or better). Allow consistent sin/cos mix only. B1 B1 DM1 can be implied by $\left(\frac{12}{13}e\right)^2 + \left(\frac{5}{13}\right)^2 = \frac{4}{13}$.
	$e = \frac{1}{4}\sqrt{3}$ ONLY	A1	AWRT 0.433 or better (0.4330127...).
		4	

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Question	Answer	Marks	Guidance
4(b)	$\tan \beta = \frac{1}{4}\sqrt{3} \times \frac{12}{5} \quad \left[\Rightarrow \tan \beta = \frac{3}{5}\sqrt{3} = 1.03923... \right]$	*B1FT	FT <i>their e</i> provided $0 < e < 1$ to find an expression for $\tan \beta$ or β . If correct, may see an angle of $46.102...$ (which may be seen on a diagram). Or for $\cos \beta = \frac{5}{26}\sqrt{13}$ or $\sin \beta = \frac{3}{26}\sqrt{39}$.
	$\tan(60 - \beta) = 0.247...$	DB1	Follow through <i>their</i> $\tan \beta$ or β (the angle between <i>P</i> and the first barrier after the first collision). If exact and correct, may see $\tan(60 - \beta) = \frac{1}{7}\sqrt{3}$. OR for finding the angle between the second barrier and <i>P</i> before the second collision – if correct, this angle is $13.897...$ May be seen on a diagram.
	$w \sin \theta = e v \sin(60 - \beta)$ $w \cos \theta = v \cos(60 - \beta)$	M1	Allow, for example, w_1 for $w \sin \theta$ and w_2 for $w \cos \theta$ – both equations must be correct. <i>Their</i> value for β does not need to be substituted for this mark. Allow <i>e</i> or <i>their</i> value for <i>e</i> . Can be implied by $\tan \theta = e \tan(60 - \beta)$.
	$\left[\tan \theta = e \tan(60 - \beta) = \frac{1}{7}\sqrt{3} \times \frac{1}{4}\sqrt{3} = \frac{3}{28} \Rightarrow \right] \theta = 6.12^\circ$	A1	AWRT 6.12 (6.1155035...) or 6.11 WWW.
		4	

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Question	Answer	Marks	Guidance
5(a)	$\frac{1}{2}m \times (2\sqrt{ag})^2 = mga(1 - \cos \theta) + \frac{1}{2}mv^2$ or $-mga + \frac{1}{2}m \times (2\sqrt{ag})^2 = -mga \cos \theta + \frac{1}{2}mv^2$ $[\Rightarrow v^2 = 2ag \cos \theta + 2ag]$	*M1	Three term energy equation (OE). Must be dimensionally correct. Allow sign errors and sin/cos mix only. Mass must be present.
	$T - mg \cos \theta = \frac{mv^2}{a}$	B1	N2L radially.
	$T - mg \cos \theta = \frac{m}{a} \times (2ag \cos \theta + 2ag)$	DM1	Combine 2 equations (N2L radially and energy). Their N2L radially can have sign errors and sin/cos mix only. Dimensionally correct.
	$[T =] mg(3 \cos \theta + 2)$	A1	OE
		4	

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Question	Answer	Marks	Guidance
5(b)	$mg(3\cos\theta + 2) = 0 \quad \left[\Rightarrow \cos\theta = -\frac{2}{3} \right]$	*M1	Setting <i>their</i> dimensionally correct expression for T from 5(a) to zero (OE from a restart, so may see $\cos\alpha = \frac{2}{3}$ being derived). <i>Their</i> expression for T must be two terms only with only one term containing either sine or cosine.
	$v^2 = 2ag \times \left(-\frac{2}{3}\right) + 2ag \quad \left[\Rightarrow v^2 = \frac{2}{3}ag \right]$	DM1	Find an expression (possibly unsimplified) for v^2 or v when the string goes slack in terms of a (and possibly g). <i>Their</i> expression for v^2 must be two terms only with only one term containing either sine or cosine.
	$v_y = \sqrt{\frac{2}{3}ag} \times \sqrt{1 - \left(-\frac{2}{3}\right)^2} \quad \left[\Rightarrow \frac{1}{9}\sqrt{30ag} \right]$	DM1	For $v\sin\theta$ following through <i>their</i> v and their value of $\cos\theta$. If correct, might see $(1.9245\dots)a$. Dependent on both previous method marks.
	$0 = \left(\frac{1}{9}\sqrt{30ag}\right)^2 + 2(-g)s_y \quad \left[\Rightarrow s_y = \frac{5}{27}a \right]$	DM1	For use of $v^2 = u^2 + 2as$ (or equivalent complete method) with $v = 0$, $a = -g$ and <i>their</i> v_y for u . If correct, might see $(0.1851\dots)a$. Dependent on all previous method marks.
	$\frac{50}{27}a$	A1	Condone $1.85a$ or better $(1.85185\dots)a$ or $\frac{5}{27}ag$. Dependent on full marks in part (a) (or a fully correct restart in this part).
		5	

Question	Answer	Marks	Guidance
6(a)	EITHER: $\frac{15x^2}{2 \times 2.5} \quad [\Rightarrow 3x^2]$	(B1)	Correct expression for the EPE.
	$1.5g(2.5 + x) = \frac{15x^2}{2 \times 2.5} + \frac{1}{2} \times 1.5 \times v^2$	M1	Conservation of energy with exactly 3 terms (GPE and KE terms must be correct). Dimensionally correct – allow sign errors and slips in the EPE term (but must be of the form $k_1 x^2$).
	$37.5 + 15x = 3x^2 + 0.75v^2$ $\Rightarrow v^2 = 50 + 20x - 4x^2 = -4(x^2 - 5x - 12.5)$	A1)	AG – at least one line of working from application of conservation of energy to given answer. Any error seen is A0 .
	OR: $1.5g - \frac{15}{2.5}x = 1.5v \frac{dv}{dx}$ $\left[\Rightarrow v \frac{dv}{dx} = 10 - 4x \right]$	(B1)	
	$\frac{1}{2}v^2 = 10x - 2x^2 + c$ $x = 0, v^2 = 2 \times 10 \times 2.5 = 50 \Rightarrow c = 25$	M1	Integrate correctly and explicitly use the correct initial condition to find c . As a minimum, must see $x = 0, v^2 = 50 \Rightarrow c = 25$ (OE, for example, may find that the constant for <i>their</i> expression is 50).
	$\frac{1}{2}v^2 = 10x - 2x^2 + 25 \Rightarrow v^2 = -4(x^2 - 5x - 12.5)$	A1)	AG Any error seen is A0 .
	Available marks	3	

Question	Answer	Marks	Guidance
6(b)	6.83 only	B1	AWRT 6.83 (6.8301270...) or $\frac{5}{2}(1 + \sqrt{3})$. If $-1.830127...$ seen it must be explicitly rejected. If working shown, then it must be correct.
		1	
6(c)	[Maximum occurs when] $x = 2.5$	B1	The correct answer for v can imply this mark. Must be clear that 2.5 is the extension at maximum speed.
	$[v^2 = 75 \Rightarrow] v = 8.66 \text{ m s}^{-1}$	B1	AWRT 8.66 (8.660254...) or exact, for example, $5\sqrt{3}$.
		2	

Question	Answer	Marks	Guidance
7	EITHER: $[s_{hor} =] 10 + 5 \times \frac{3}{5} \quad [\Rightarrow 13] \text{ and } [s_{ver} =] \frac{4}{5} \times 5 \quad [\Rightarrow 4]$	(*B1)	For both unsimplified distances – may see these two values on a diagram.
	$4 = ut \sin \theta - \frac{1}{2}gt^2$ and $13 = ut \cos \theta$	DM1	Applying $s = ut + \frac{1}{2}at^2$ both horizontally and vertically. Allow sin/cos mix only. No sign errors, must be using $a = -g$. Dependent on previous B1 mark.
	$\frac{u \cos \theta}{gt - u \sin \theta} = \frac{4}{3}$ or $\frac{u \sin \theta - gt}{u \cos \theta} = -\frac{3}{4}$ $[\Rightarrow -3u \cos \theta = 4u \sin \theta - 4gt]$	B1	Use perpendicular condition to form a correct equation in u , t and θ .
	$-3\left(\frac{13}{t}\right) = 4\left(\frac{4 + \frac{1}{2}gt^2}{t}\right) - 4gt \quad [\Rightarrow 2gt^2 = 55]$	M1	Eliminate two of u, θ, t to obtain an equation in one variable only – dependent on all previous marks . Can be implied by $t = \frac{1}{2}\sqrt{11} = 1.65831\dots$
	$[u \cos \theta = \frac{26}{11}\sqrt{11}, u \sin \theta = \frac{71}{22}\sqrt{11} \Rightarrow] \quad \theta = 53.8^\circ$	A1	AWRT 53.8 (53.78116...) or 0.939 (0.938658...).
	$u = 13.3$	A1)	AWRT 13.3 (13.26735...) or $\sqrt{\frac{7745}{44}}$.

Question	Answer	Marks	Guidance
7	OR: Alternative Method 1 $[s_{hor} =] 10 + 5 \times \frac{3}{5} \quad [\Rightarrow 13] \text{ and } [s_{ver} =] \frac{4}{5} \times 5 \quad [\Rightarrow 4]$	(*B1)	For both unsimplified distances – may see these two values on a diagram.
	$4 = 13 \tan \theta - \frac{g \times 13^2}{2u^2} (1 + \tan^2 \theta)$	DB1	Correct trajectory equation in terms of u and θ in any form.
	$\tan \theta - \frac{g \times 13}{u^2} (1 + \tan^2 \theta) = -\frac{3}{4}$ $[\Rightarrow 4u^2 \tan \theta - 4g \times 13 \times (1 + \tan^2 \theta) = -3u^2]$ or $\frac{4}{3} = \frac{u \cos \theta}{\sqrt{u^2 \sin^2 \theta - 2g \times 4}} \text{ or } -\frac{3}{4} = \frac{u \sin \theta - \frac{13g}{u \cos \theta}}{u \cos \theta}$	M1	Differentiate general trajectory equation, set equal to $-\frac{3}{4}$ and use <i>their</i> horizontal distance travelled to form an equation in u and θ only. Allow a single slip in the differentiation of the trajectory equation. OR Use perpendicular condition to form a correct equation in u and θ .
	$\frac{169g(1 + \tan^2 \theta)}{26 \tan \theta - 8} = \frac{4g \times 13(1 + \tan^2 \theta)}{4 \tan \theta + 3}$	M1	Attempt to combine <i>their</i> two equations to arrive at an equation in either u or θ only – dependent on all previous marks . May see, if correct, $\frac{169}{16} \tan^2 \theta - 13 \tan \theta - \frac{497}{256} [= 0]$.
	$[\tan \theta = \frac{71}{52} \Rightarrow] \theta = 53.8^\circ$	A1	AWRT 53.8 (53.78116...) or 0.939 (0.938658...).
	$u = 13.3$	A1)	AWRT 13.3 (13.26735...) or $\sqrt{\frac{7745}{44}}$.

Question	Answer	Marks	Guidance
7	OR: Alternative Method 2 $[s_{hor} =] 10 + 5 \times \frac{3}{5} \quad [\Rightarrow 13] \text{ and } [s_{ver} =] \frac{4}{5} \times 5 \quad [\Rightarrow 4]$	(B1)	For both unsimplified distances – may see these two values on a diagram.
	$\left(\frac{5u \cos \theta}{4} \times \frac{3}{5}\right)^2 = u^2 \sin^2 \theta - 2 \times 10 \times 4 \quad [\Rightarrow \frac{9}{16} u^2 \cos^2 \theta = u^2 \sin^2 \theta - 80]$	M1	M1 for at least one equation in terms of u and θ only. Allow sin/cos mix on the θ terms only (no other errors allowed).
	$13 = u \cos \theta \times \left(\frac{5u \cos \theta}{4} \times \frac{3}{50} + \frac{u \sin \theta}{10}\right) \quad \left[\Rightarrow 13 = \frac{u^2 \cos \theta}{40} (3 \cos \theta + 4 \sin \theta)\right]$	A1	For two different correct equations in u and θ only. Might see $-\frac{3}{5} \times \frac{5}{4} u \cos \theta = u \sin \theta - 10 \times \frac{13}{u \cos \theta}$.
	$-80 = \frac{13}{\cos \theta} \left(\frac{40}{3 \cos \theta + 4 \sin \theta}\right) \left(\frac{9}{16} \cos^2 \theta - \sin^2 \theta\right)$	M1	Attempt to combine <i>their</i> two equations to arrive at an equation in either u or θ only – dependent on all previous marks .
	$\left[\tan \theta = \frac{71}{52} \Rightarrow\right] \theta = 53.8^\circ$	A1	AWRT 53.8 (53.78116...) or 0.939 (0.938658...).
	$u = 13.3$	A1)	AWRT 13.3 (13.26735...) or $\sqrt{\frac{7745}{44}}$.
	Available marks	6	